First Order Low and High Pass Passive Filters

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First Order Low Pass Filter

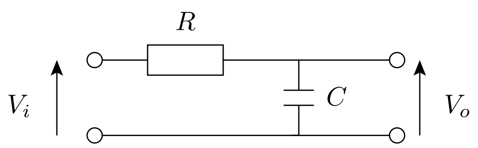

Figure 1: First order low pass filter. The current I_1 flows through the resistor and capacitor, oriented in the clockwise direction.

The equations of the circuit and transfer function are:

$$
\begin{aligned}
V_i - I_1 R - I_1 X_C &= 0 && (1)\\
I_1 X_C &= V_o \quad \therefore && (2)\\
I_1 &= \frac{V_o}{X_C} \quad \therefore && (3)\\
V_i - \frac{V_o}{X_C} R - \frac{V_o}{X_C} X_C &= 0 \quad \therefore && (4)\\
V_i - V_o \frac{(R - X_C)}{X_C} &= 0 \quad \therefore && (5)\\
\frac{V_o}{V_i} &= \frac{X_C}{R - X_C} && (6)
\end{aligned}
$$

For the s-domain description, recall that:

$$X_C = \frac{1}{sC} \tag{7}$$

which can be substituted into (6):

$$
\begin{aligned}
\frac{V_o}{V_i} &= \frac{X_C}{R - X_C} && (8)\\
&= \frac{1}{sC(R - \frac{1}{sC})} && (9)\\
&= \frac{1}{sCR - 1} && (10)
\end{aligned}
$$

To obtain the frequency response we need the magnitude of the transfer function at $s = j\omega$,

$$\frac{V_o}{V_i} = \frac{1}{-1 + j\omega CR} \tag{11}$$

$$= \frac{-1 - j\omega CR}{(-1)^2 + (\omega CR)^2} \tag{12}$$

$$= \frac{-1}{1 + \omega^2 C^2 R^2} - \frac{j\omega CR}{1 + \omega^2 C^2 R^2} \tag{13}$$

the magnitude of this is obtained as follows,

$$\left|\frac{V_o}{V_i}\right|^2 = \left(\frac{-1}{1 + \omega^2 C^2 R^2}\right)^2 + \left(\frac{\omega CR}{1 + \omega^2 C^2 R^2}\right)^2 \tag{14}$$

$$= \frac{1 + \omega^2 C^2 R^2}{(1 + \omega^2 C^2 R^2)^2} \tag{15}$$

$$= \frac{1}{1 + \omega^2 C^2 R^2} \tag{16}$$

finally, taking the square root,

$$\left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{1 + \omega^2 C^2 R^2}}. \tag{17}$$

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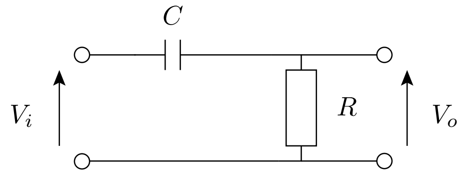

Figure 2: First order high pass filter. The current I_1 flows through the capacitor and resistor, oriented in the clockwise direction.

The equations of the circuit and transfer function are:

$$V_i - I_1 X_C - I_1 R = 0 \tag{18}$$

$$I_1 R = V_o \quad \therefore \tag{19}$$

$$I_1 = \frac{V_o}{R} \quad \therefore \tag{20}$$

$$V_i - \frac{V_o}{R}(X_C + R) = 0 \quad \therefore \tag{21}$$

$$V_i = \frac{V_o}{R}(X_C + R) \quad \therefore \tag{22}$$

$$\frac{V_o}{V_i} = \frac{R}{X_C + R} \tag{23}$$

For the s-domain description, recall that:

$$X_C = \frac{1}{sC} \tag{24}$$

which can be substituted into (23):

$$\frac{V_o}{V_i} = \frac{R}{X_C + R} \tag{25}$$
$$= \frac{R}{\frac{1}{sC} + R} \tag{26}$$
$$= \frac{sCR}{1 + sCR} \tag{27}$$

To obtain the frequency response we need the magnitude of the transfer function at $s = j\omega$,

$$\frac{V_o}{V_i} = \frac{j\omega CR}{1 + j\omega CR} \tag{28}$$
$$= \frac{j\omega CR(1 - j\omega CR)}{1^2 + \omega^2 C^2 R^2} \tag{29}$$
$$= \frac{j\omega CR + \omega^2 C^2 R^2}{1 + \omega^2 C^2 R^2} \tag{30}$$
$$= \frac{\omega^2 C^2 R^2}{1 + \omega^2 C^2 R^2} + j\frac{\omega CR}{1 + \omega^2 C^2 R^2} \tag{31}$$

the magnitude of this is obtained as follows,

$$\left|\frac{V_o}{V_i}\right|^2 = \frac{(\omega^2 C^2 R^2)^2 + (\omega CR)^2}{(1 + \omega^2 C^2 R^2)^2} \tag{32}$$
$$= \frac{(\omega CR)^4 + (\omega CR)^2}{(1 + \omega^2 C^2 R^2)^2} \tag{33}$$
$$= \frac{(\omega CR)^2(1 + (\omega CR)^2)}{(1 + \omega^2 C^2 R^2)^2} \tag{34}$$
$$= \frac{(\omega CR)^2}{(1 + \omega^2 C^2 R^2)} \tag{35}$$

finally, taking the square root,

$$\left|\frac{V_o}{V_i}\right| = \frac{\omega CR}{\sqrt{1 + \omega^2 C^2 R^2}}. \tag{36}$$